

Abstract relational thinking ability of high school students through the rigorous mathematical thinking (RMT) approach to algebra topics

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Abstract

Algebra teaching at RMT is designed to involve students in consciously and deliberately practicing the formation of conceptual elements from mathematical functions. This research is a qualitative descriptive study which aims to see the emergence of indicators on levels think relational abstract with use approach RMT on the topic of algebra, material on Systems of Linear Equations with Three Variables. The research subjects in this study were 26 students of SMA in Palembang class Implementation procedures study consists from three stages that is stage preparation, stage implementation, and data analysis stage. The research was conducted in three meetings, the first meeting and second done in a way offline using the RMT approach, as well as meeting third he gave question test abstract relational abilities online through the zoom meetings application. The results of research obtained at SMA in Palembang class well, based on students being able to reach indicators of the level of abstract relational thinking.

Keywords : Algebra, Rational Thinking, RMT, SPLTV

Abstrak

Pengajaran aljabar pada RMT dirancang untuk melibatkan siswa secara sadar dan sengaja mempraktikkan pembentukan elemen konseptual dari fungsi matematika. Penelitian ini merupakan penelitian deskriptif kualitatif yang bertujuan untuk melihat kemunculan indikator pada level berpikir relasional abstrak dengan menggunakan pendekatan RMT pada topik aljabar materi Sistem Persamaan Linear Tiga Variabel. Subjek penelitian pada penelitian ini yakni siswa SMA di Palembang kelas X Olimpiade sebanyak 26 siswa, sedangkan yang menjadi fokus subjek penelitian yaitu 4 orang siswa dengan 2 siswa berkemampuan tinggi dan 2 siswa berkemampuan sedang secara *purposive sampling* atas rekomendasi guru. Prosedur pelaksanaan penelitian terdiri dari tiga tahapan yaitu tahap persiapan, tahap pelaksanaan, dan tahap analisis data. Penelitian dilakukan sebanyak tiga kali pertemuan, pertemuan pertama dan kedua dilakukan secara *offline* dengan menggunakan pendekatan RMT, serta pertemuan ketiga diberikannya soal tes kemampuan relasional abstrak secara *online* melalui aplikasi *zoom meetings*. Hasil penelitian yang diperoleh pada SMA di Palembang kelas X Olimpiade bahwa kemampuan relasional abstrak siswa melalui pendekatan RMT pada topik aljabar sudah sangat baik, berdasarkan siswa sudah dapat mampu mencapai indikator dari level berpikir relasional abstrak.

Kata kunci: Aljabar, Berpikir Rasional, RMT, SPLTV

INTRODUCTION

Mathematics is a study of structure, space, quantity and change (Sukmawati, 2019). One of the important ability learned in mathematics is the ability to think abstractly. Abstract relational thinking, can help students to activate previous knowledge related to mathematics, provide and articulate mathematical logical evidence, define problems, inferential-hypothetical thinking, project and restructure relationships, form proportional quantitative relationships, form a functional relationship, form a unified relationship functional, mathematical inductive-deductive thinking, mathematical analogical thinking, mathematical syllogistic thinking, transitive mathematical relational thinking, and outlining mathematical activities through category cognitive (Kinard & Alex, 2012). Abstract relational thinking is needed in solving algebra problems because the topic of algebra leads to students' experiences in understanding a concept, procedure and its application by solving it using various strategies, so that abstract thinking skills are needed to connect various concepts without real problems, objects or ideas (Nihayah, 2021). Middle school students (approximately 12-18 years old) have focused on forming and operating on polynomial and relational expressions, representing word problems with algebraic expressions and equations. contains known and unknown variables, and solves algebraic equations through axiomatic properties and equivalence (Kieran, 2018).

However, in reality students still experience difficulties in understanding concepts and solving algebra problems (Herawati & Kadarisma, 2021; Nugraha & Zanthi, 2019; Nurikawai, Sagita, & Setiyani, 2021) . One of difficulties students have is that they intend to be less careful in algebraic calculations and less understanding of material concepts (Ramadhani & Firmansyah, 2021). In the three-variable linear equation system (SPLTV) material, students still make some significant mistakes, such as errors in transforming problems, processing skills, and writing final answers (Baskorowati, 2021; Dewi & Kartini, 2021; Kuswanti, Sudirman, & Nusantara, 2018). Therefore, teaching innovations are required in providing algebra learning to students, such as using learning approaches or models (Mulyani, Indah, & Satria, 2018).

An approach called *Rigorous Mathematical Thinking* (RMT) can be the solution of these problems. RMT is an approach which based on two main theories, namely Vygotsky's theory of psychological tool theory and Feuerstein's theory about *Mediated Learning Experience* (MLE) which where second theory about the cognitive function (Kinard & Alex, 2012). RMT consists of from three levels, qualitative thinking, quantitative thinking, and abstract relational thinking (Kinard & Alex, 2012). In designing learning activities using the RMT approach can be an effective learning alternative (Hidayat, Kohar, Prihartiwi, Mubarak, & Yohannes, 2021). Also, RMT can create good

interactions with students during the process learning taking place. RMT is useful for facilitating students to represent visual in-depth understanding by providing practice questions and build strong cognitive process and build mathematical concepts (Firmasari & Santi, 2019). RMT can also be used to develop thinking skill in which students have to form understanding, change and make conclusion, understanding logical idea as foundation to obtained solution for solving problem as well as get new knowledge (Meilantifa & Budiarto, 2018).

The results of the research showed that students' PKM abilities in RMT approach learning were higher than direct approach learning (Hendrayana, 2017). This can be seen from the female gender factor being greater than the male gender in using the RMT approach. Other research from (Ifana, Aisyah, Pratiwi, Kurniadi, & Araiku, 2021) shows that many algebra topics are done well, namely labeling, systematic searching to collect and complete information, comparison, use of more than one source, encoding and code breaking. In the research carried out, researchers will produce abstract rational thinking abilities that have 10 indicators that are in accordance with RMT learning by developing abilities think Which students have to form understanding, change and make conclusion, helpful for understand draft and idea which logical as well as plan use idea which obtained for solve problem so students will get knowledge new from abstract rational thinking.

The application of RMT focuses on mediating students in building mathematical concepts using three phases, namely cognitive development, content as process development, cognitive conceptual construction practice *accompanied* by six process steps (Yulianto, 2021). These processes do not occur linearly, but each phase and step is important for student involvement in conceptual understanding of mathematics. The RMT bond involves cognitive, affective and conceptual dimensions. So that in implementing the RMT approach in the learning process it will be related to the ability to think abstractly, because the cognitive development process requires cognitive development abilities.

Based on this description, this research aims to see high school students' abstract relational thinking abilities through the RMT approach on algebra topics, so researchers are interested in conduct research Which title "Ability Think Relational Abstract Student High School Through *Rigorous Mathematical Thinking* (RMT) Approach on Algebra Topics".

RESEARCH METHODS

This research uses a descriptive type with a qualitative approach which aims to describe students' abstract relational abilities through the RMT approach to algebra topics. The subjects in this research were class subject study that is SA and MRR with ability academic tall and AN and NSR with academic abilities currently. These 4 students will work on questions that have been validated by validators at the learning evaluation stage to see abstract relational thinking abilities.

Students' abstract relational thinking abilities can be seen from the test results which consist of 2 descriptive questions. This question was chosen because in doing abstract relational thinking the process is carried out online and the difficulty of the question given. Therefore, researchers only gave 2 questions to measure the ability to think abstractly. Meanwhile, in the abstract relational indicators, there are 10 indicators containing abstract relational abilities chosen by researchers in the research. Indicators and descriptors can be seen in **Table 1** below:

Table 1. Indicators of abstract realistic thinking level

No	Indicator	Descriptor
1.	Previous mathematical knowledge is activated	Gathering student knowledge previously so that can connected and adjusted to what you think with previous experience.
2.	Defining the problem	Examining existing problems by analyzing and looking for relationships to find out what steps are needed will be done.
3.	Hypothetical thinking	Outlining temporary assumptions, then looking for evidence to help or contrary to expectations.
4.	Establishment of quantitative relationships	Make quantitative relationships between one concept and another by looking for the relationship between the two concepts.
5.	Form functional relationships	Creating a relationship between two or more things whose value changes so that it is necessary one another.
6.	Form functional relationships units	Create a relationship between the change in the amount of the dependent variable produced by a unit change in the amount to the independent variable defined by the functional relationship between the two variables expressed in terms of a mathematical function or algebraic equations.
7.	Inductive deductive Thinking mathematics	Taking aspects of various mathematical details that seem to form patterns, categorize them into relationships, organize the results to form general mathematical rules, principles, formulas, recipes, or guides, and use formulas for certain situations
8.	Analogical thinking mathematics	Understandable structures well and mathematical operations new, principles, or issues, form the relational aspects of the components of each structure separately. Using one's knowledge about a situation that can be understood with Good thus building understanding new.
9.	Relational thinking	Make a mathematical conjecture that will later show a relationship between the concept of P and the concept of Q with a second conjecture that provides a link between the concept of P and the concept of R, then conclude the relationship between the Q concept and the R concept.
10.	Describe the activity mathematics	Understand and analyze mathematical activities

The subjects in this research were class X students of SMA in Palembang 2021/2022 academic year. The time to conduct this research is from August 2021 to October 2021 at SMA in Palembang. Students were selected based on recommendations from Mathematics teachers at school taking into account the objectives of this research. The research implementation procedure consists of three stages, namely the preparation stage, implementation stage, and data analysis stage. The research was conducted in three meetings, the first and second meetings were conducted offline, and at the third meeting, abstract relational ability test questions were given online via the *zoom application meetings*. Technique collection data on study this is test, studies documentation, and interview. On data test done for see indicator which appear on student answers. The test questions consist of 2 questions that contain abstract relational indicators, while the interview can support data from the test results that students have completed. Research data which has collected will analyzed in a way qualitative with describe whether or not indicators of literacy ability appear mathematical.

In this research, we will use the Miles and Huberman data analysis technique, where the qualitative data analysis technique is carried out in 3 stages, namely data reduction, data presentation, and drawing conclusions.

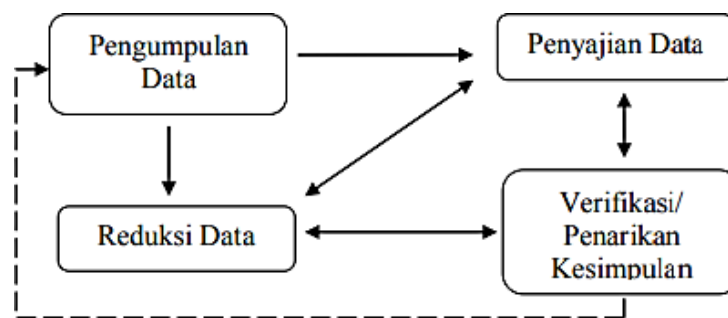


Figure 1 Miles and Huberman cycle model

RESULTS AND DISCUSSION

In the results and discussion of this research, the researcher will describe the results obtained when conducting research on the abstract relational abilities of high school students on the topic of algebra. Before conducting the research, the researcher had prepared various research instruments needed, such as RPP, LKPD, test questions, and interview guidelines, as well as determining the research subjects at SMA in 1 Palembang. The instrument was validated by lecturers and teachers at the school. The

following are questions presented by researchers to determine students' abstract relational abilities:

Question:

1. Currently, the Korean music industry is in great demand by most people in the world, making shares of several *entertainment companies* soar. There are 3 large music industry companies in Korea, namely *SM Entertainment* , *JYP Entertainment* , and *YG Entertainment* . These three companies are currently experiencing an increase in their shares comparison share BC And JYP that is 2 : 3. Whereas comparison The increase in JYP and YG shares is 6: 5. And we know that the increase in JYP shares is greater than SM shares. If the total increase in SM and JYP shares is 20% more than the increase in shares WHICH?

Look for: a) Increase in shares WHICH, b) Increase in shares JYP
c) Increase in shares BC

2. In Konoha Village there are 3 great ninjas, namely Naruto, Sakura and Sasuke. They were put together to do ninja training in academics with teacher Kakashi. Teacher Kakashi give they task For look for roll, when Naruto And Sakura together can found it during 4 O'clock, Sakura And Sasuke working together can find it for 3 hours, while Naruto and Sasuke working together can find it during 2,4 O'clock. In How many O'clock they can find the reels, if they work individually?

In this research, subjects were selected using *purposive sampling* as described in the research method. Researchers asked mathematics teachers to recommend students who were suitable to be used as research subjects with the criteria of high and moderate academic ability. The mathematics teacher recommends class subject study that is S.A And MRR with ability academic tall And AN and NSR with academic abilities currently. The selection of this ability is only high and medium ability because if students with low ability work on the evaluation questions given with low student mastery of mathematics subject matter, of course the student will find it difficult (Zumala, 2019). So students with low abilities were not chosen as research subjects.

From fourth subject the, subject S.A has fulfil all indicator predefined abstract relational. From the results of the answers, subjects have been able to gather previous mathematical knowledge to connect and adapt aspects that have been thought about from previous experience. This is indicated by the subject's answer being able to find out the solution to finding variables using algebraic procedures.

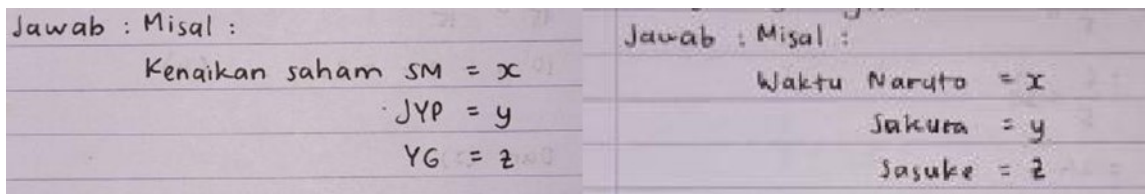


Figure 2. SA Subject's answers in defining the problem

In Figure 2, the SA subject is skilled at defining problems, this is indicated by the SA subject being able to correctly define variables in the context of the existing problem.

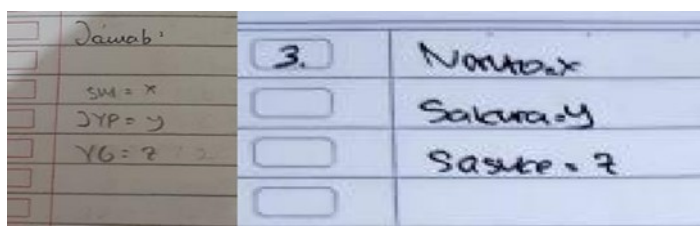


Figure 3. Representative responses of subjects who are wrong in defining the problem

However, subject MRR, AN, And NSR Still wrong in define problem, This is because the subject considers the variables to be inaccurate, which can be seen in **Figure 3**.

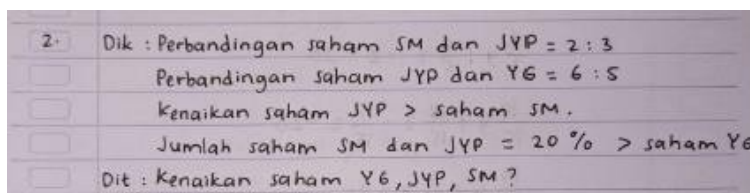
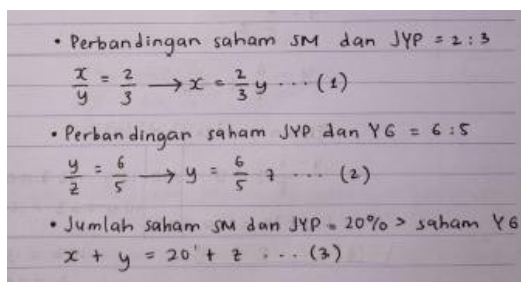


Figure 4. Representative SA subject answers in hypothetical thinking

Furthermore, SA subjects were also able to think hypothetically by writing the answer "Increase share JYP > share BC." However, subject MRR, AN, And NSR No write down hypothetical answers that can help the subject in solving problem.



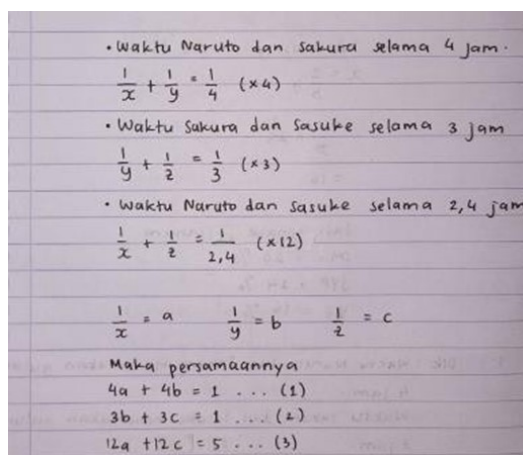
• Perbandingan saham SM dan JYP = 2 : 3
 $\frac{x}{y} = \frac{2}{3} \rightarrow x = \frac{2}{3}y \dots (1)$

• Perbandingan saham JYP dan YG = 6 : 5
 $\frac{y}{z} = \frac{6}{5} \rightarrow y = \frac{6}{5}z \dots (2)$

• Jumlah saham SM dan JYP = 20% > saham YG
 $x + y = 20 + z \dots (3)$

Figure 5. Representative of subject answers in establishing proportional quantitative relationships and forming functional relationships

In **Figure 5**, the subject is able to make quantitative connections between one concept and another by looking for the relationship between the two concepts. And also the subject is able to make connections between two or more things whose values change so that required one the same other. This matter be marked on answer subject which can link the concept of comparison and the form of algebraic equations by changing the form of comparison to equation form algebra.



• Waktu Naruto dan sakura selama 4 jam.
 $\frac{1}{x} + \frac{1}{y} = \frac{1}{4} \quad (\times 4)$

• Waktu sakura dan sasuke selama 3 jam
 $\frac{1}{y} + \frac{1}{z} = \frac{1}{3} \quad (\times 3)$

• Waktu Naruto dan sasuke selama 2,4 jam
 $\frac{1}{x} + \frac{1}{z} = \frac{1}{2,4} \quad (\times 12)$

$\frac{1}{x} = a \quad \frac{1}{y} = b \quad \frac{1}{z} = c$

Maka persamaannya
 $4a + 4b = 1 \dots (1)$
 $3b + 3c = 1 \dots (2)$
 $12a + 12c = 5 \dots (3)$

Figure 6. Representative subject answers in forming functional unit relationships and mathematical inductive deductive thinking

In **Figure 6**, the subject can form unit functional relationships and can carry out mathematical inductive deductive thinking. This can be seen from the answers of students who are able to make algebraic equations from problems relating the number of people to the time required (inverse value comparison). Students also make the solution procedure easier by converting fractions into variable forms.

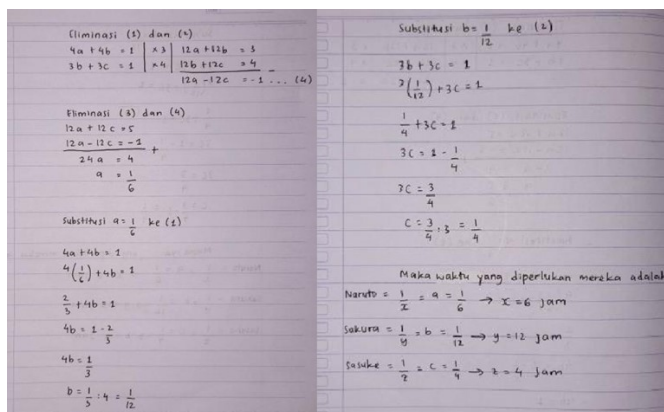


Figure 7. Representative Subject Answers in Mathematical Analogical Thinking and Relational Thinking

In **Figure 7**, the subject can use his knowledge about a situation to build a new understanding and can make mathematical conjectures which will later show a relationship between the concept of P and the concept of Q with the second conjecture providing linkages between draft P And draft R, Then conclude connection between the Q concept and the R concept. This is indicated by the subject's answer, where the subject can apply the concept of inverse comparison of values with the concept of algebraic equations. Subjects also simplify their comparisons by making comparisons with a variable, so that the subject can easily solve the problem given.

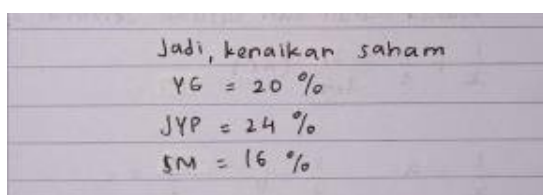


Figure 8. Representative responses of subjects in describing mathematical activities with cognitive indicators

In **Figure 8**, it can be seen that the subject has understood and analyzed mathematical activities from problem which given. This matter be marked with results answer a subject that provides conclusions from the problems given appropriate.

Learning using the *Rigorous Mathematical Thinking* (RMT) approach is better than using the conventional approach. This is proven by the research results that students are able to solve questions to measure abstract relational thinking abilities. So research using the RMT approach has an influence on the abstract relational abilities of high school students in SPLTV lessons. (Yulianto, 2021) in his study described that one of the

goals of mathematics education is to make students think creatively. The result of creative thinking is called creativity. However, in reality, this creativity is not paid enough attention by teachers in learning. So creative thinking abilities need to be developed further. Providing stimulus is one way to develop creative thinking skills by paying attention to students' habit of mind level through the rigorous mathematical thinking approach. This research is experimental research with quasi experimental design and uses a 3 x 2 factorial design. The population in this research is class VII MTs students. The sample used was 2 classes, where 1 class was the experimental class and 1 class was the control class. The instruments used were mathematical creative thinking ability tests, habits of mind attitude scales, observations and interviews. The data used to test the difference between two means is the t-test, two-way ANOVA and correlation. The results of this research show that 1) there is a difference in the effect of implementing rigorous mathematical thinking approach learning and conventional approaches on increasing students' mathematical creative thinking abilities, 2) there is no difference in the influence of the level of habit of mind on increasing students' mathematical creative thinking abilities, 3) there is no effect interaction of learning approach and level of habit of mind towards increasing students' mathematical creative thinking abilities, 4) there is no positive correlation between students' mathematical creative thinking abilities and level of habit of mind, the effect of RMT learning on creative thinking skills with higher results than the conventional approach, however, in this research there was no interaction effect in the learning approach applied in the experimental class or control class on increasing mathematical creative thinking skills each student. Meanwhile, research conducted by researchers, in measuring abstract relational thinking abilities, has better results than the conventional approach and has an influence on the interaction of the RMT approach in the learning process, so that students not only have higher results than the conventional approach, but students can also interact in the learning process carry out the learning process.

From the test results and analysis, it was found that the abstract relational abilities of students with medium and high academic abilities were said to be very good. Most students were able to understand and solve the problems given, although there were still some students who were wrong in defining the problem.

CONCLUSION

Based on the research results obtained at SMA in Palembang class with the 2 description questions given, students are able to solve problems well, so they can reach indicators of the level of abstract relational thinking. The mistakes that are seen are when students are not careful in identifying variables in the context of the problem given. For this reason, students must continue to be regularly given questions to hone their abstract relational thinking skills.

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