Jurnal Pendidikan Matematika RAFA
p-ISSN :2460-8718
e-ISSN : 2460-8726

Available online at:
http://jurnal.radenfatah.ac.id/index.php/jpmrafa
June 2025, 11(1): 1-15

Analysis of first-year math teacher students' thinking on the birthday cake problem

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(Received 06-10-2024, Reviewed 07-01-2025, Accepted 30-01-2025)

Abstract

The ultimate aim of this learning process is that students will be able to conduct their own mathematical investigations and identify their mistakes.This research aims to: identify the mathematical concepts used by prospective mathematics teacher students in solving birthday cake problem and describe the flow of the mathematical thinking process of first -year prospective mathematics teacher students in solving birthday cake problem. The subjects of this research were second-semester students in the Mathematics Education Study Program at Bengkulu University, totaling 60 students. Data was collected using tests and interviews. Students were given a mathematical thinking process test in the form of a birthday cake context problem. The data was analyzed by grouping test results based on indicators of the mathematical thinking process. To gain deeper insights, students representing the answers using the concepts of number patterns, combinations, and series were interviewed. The results of the research show that: students used three mathematical concepts in solving the birthday cake problem, namely the concept of number patterns, combinations, and arithmetic series addition; and the flow of the mathematical thinking process used by students included visualizing the birthday cake problems, then generalizing and proving their solutions. This research implies that developing test instruments should guide students in forming diverse mathematical thinking processes by integrating various concepts in their solutions, thereby enhancing their mathematical thinking abilities.

Keywords: *Birthday Cake Problem, Mathematical Thinking Process*

Abstrak

Penelitian ini bertujuan agar mahasiswa mampu melakukan penyelidikan matematis sendiri dan mampu mengidentifikasi dimana letak kesalahan yang dilakukannya. Penelitian ini bertujuan untuk: mengetahui konsep matematika apa saja yang digunakan mahasiswa calon guru matematika dalam menyelesaikan permasalahan kue ulang tahun dan mendeskripsikan gambaran alur proses berpikir matematis mahasiswa calon guru matematika tahun pertama dalam menyelesaikan permasalahan kue ulang tahun. Subjek dalam penelitian ini adalah mahasiswa semester II Program Studi Pendidikan Matematika Universitas Bengkulu yang berjumlah sebanyak 60 mahasiswa. Data dikumpulkan dengan menggunakan tes dan wawancara. Mahasiswa diberikan tes proses berpikir matematis berupa soal konteks kue ulang tahun. Data dianalisis dengan cara hasil tes dikelompokkan berdasarkan indikator proses berpikir matematis. Untuk mendapatkan informasi lebih dalam, setiap mahasiswa yang mewakili jawaban menggunakan konsep pola bilangan, kombinasi dan konsep deret dilakukan wawancara. Hasil penelitian menunjukkan diperoleh cara penggunaan tiga konsep matematika yang digunakan mahasiswa dalam menyelesaikan permasalahan kue ulang tahun tersebut, yaitu konsep pola bilangan, konsep kombinasi dan konsep penjumlahan deret aritmatika. Kemudian alur proses berpikir matematis yang digunakan mahasiswa dengan menyertakan gambar dari permasalahan kue ulang tahun, kemudian melakukan generalisasi dan pembuktian. Penelitian ini memberikan implikasi agar dalam mengembangkan instrumen tes dapat mengarahkan mahasiswa membentuk alur berpikir matematis yang beragam dengan mengintegrasikan berbagai konsep dalam penyelesaiannya, sehingga mampu mengembangkan kemampuan berpikir matematis mahasiswa.

Kata kunci: Permasalahan Kue Ulang Tahun, Proses Berpikir Matematis

INTRODUCTION

The current implementation of mathematics learning must provide wide space for movement, provide understanding of the process and bring meaning to the mathematical concepts being studied. This is included in 21st century learning, where the learning carried out is expected to be able to develop students' critical thinking, collaboration, creativity and communication skills, especially prospective mathematics teachers (Ulya & Rahayu, 2021). For this reason, in implementing learning, we must consider the needs of 21st-century learning, which cannot be separated from mathematical thinking processes. The mathematical thinking process is mathematics as an activity carried out in the human mind using abstraction, symbolic representation, and symbolic manipulation, as well as the ability to communicate problems in the form of mathematical ideas so that a problem-solving process is carried out (Aminah & Firmasari, 2017; Stacey, 2007). In line with the idea that the implementation of mathematics learning in the classroom can be done by imitating a mathematician (Breen & O’Shea, 2010), mathematicians solve mathematical problems by carrying out activities that include giving examples, specializing, changing, varying, reversing, generalizing, explaining, confirming, verifying, convincing, and denying. These activities are activities that ultimately build mathematical thinking skills (Delima et al., 2018; Mason et al., 2004).

There are four basic processes in constructing mathematical thinking (Mason et al., 2010), namely: specialization is a thinking activity carried out by completing various exercises by looking at examples; generalization is an activity carried out by identifying patterns or relationships; guessing is an activity carried out by predicting correlations and results; and convincing is the activity of finding and communicating the reasons why something is considered true. Through the basic process of mathematical thinking, it is believed to provide heuristic strategic experience or the use of concepts in solving problems (Stacey, 2007), so that the mathematical thinking process can increase understanding of mathematical concepts (Mason et al., 2010; Sari, 2021). Thinking is an activity that forms a flow of understanding mathematics material involving the components of induction, deduction, clarification, and reasoning, as well as the ability to carry out analysis so as to form a conclusion based on existing inferences (Arends, 2008).

For this reason, it is necessary to develop student’s mathematical thinking abilities, especially prospective mathematics teachers. Later, student teachers in implementing learning, must be able to train students to develop mathematical thinking processes.

The implementation of mathematics learning emphasizes thinking processes, abilities, attitudes, curiosity, and being able to enjoy learning mathematics. So that it can direct students to carry out mathematical thinking processes. The thinking process is closely related to how to process and organize information in cognitive activities (Izzatin et al., 2020). The results of previous research have made a good contribution to the mathematical thinking process. Research conducted by Izzatin et al., (2020); Scusa & CO, (2008) provide good results in mathematical thinking through teacher involvement throughout learning with mathematical thinking and responding to students in a good mathematical way. Apart from that, the mathematical thinking process must also be supported by the ability to explore mathematical concepts. In fact, it is still found that mathematics learning has not been effective in developing concept exploration abilities and thinking abilities (Andriani et al., 2016; Ersoy & Güner, 2015). The ability to solve high-level mathematical problems only provides results for a small number of students (Stacey, 2007). Other research shows that many students do not know where to start to solve the mathematical problems found (Scusa & CO, 2008), and they also find low ability to interpret and link concepts in solving problems (Ulya & Rahayu, 2021).

The low ability to explore concepts and provide various heuristic solutions to a problem can have an impact on the ability to carry out mathematical thinking processes. In fact, difficulties in solving mathematical problems result in the ability to think mathematically not being developed optimally (Nuryanti, 2022). It will have a direct impact on low student learning outcomes. However, no research evidence has been found to show this. Therefore, it is necessary to study and investigate: 1) the involvement of several mathematical concepts possessed by students in solving a problem; and 2) the mathematical thinking process carried out by students in solving a problem. One problem that will be addressed is the problem of birthday cakes. The birthday cake problem is a problem whose solution can involve various mathematical concepts. By using this birthday cake problem, the researcher will get a picture of the mastery of mathematical concepts possessed by prospective first-year mathematics teacher students.

The results of the interview provide information that assessing mathematical thinking processes in the implementation of learning for students in mathematics education study programs has never been carried out. So far, assessments have been carried out only to measure abilities that prioritize understanding concepts. The form of resolution for conceptual understanding problems is limited to routine procedural solutions (Kahirunnisa et al., 2022). This condition shows that mathematical thinking process abilities have not received good attention by lecturers in mathematics education study programs. As a result of observations made during the learning process, information was obtained that there were difficulties for students in determining the Initial steps to solve problems and difficulties in determining what mathematical concepts would be used to solve these problems. Another problem was also found: students had difficulty developing ideas and communicating them as a way to determine problem solving. This condition occurs because students are not used to thinking mathematically systematically and still find difficulties in understanding the problems given, so they experience obstacles in connecting and exploring the various concepts they already have to determine various ways to solve the problems given. This condition results in low mathematical thinking abilities and non-optimal learning outcomes.

Mathematical thinking processes are carried out when the learning process is carried out. The mathematical thinking process requires practice and is carried out periodically or continuously so that students feel used to it and can develop their mathematical thinking abilities (Reviandari, 2004). As a result, an assessment of the mathematical thinking process can be carried out. Indicators used to measure mathematical thinking processes include specialization, generalization, guessing, and convincing (Stacey, 2007). Measuring the mathematical thinking process will also have a positive impact on students with low abilities. Each student is given the opportunity to discuss various concepts and combine various strategies to understand the problem and determine the solution.

Based on the existing problems, it is necessary to explore what concepts are used to solve one problem, namely the birthday cake problem. Furthermore, after discovering the concepts used by students in solving problems, it is necessary to explore in more depth the processes used by students in developing mathematical thinking abilities and describe more clearly the mathematical thinking processes carried out by students of the Mathematics Education Study Program in solving problems. This investigation of the mathematical thinking process is a form of lecture’s effort to find obstacles that exist in students and improve student's mathematical thinking processes. The aims of this research are: 1) to find out how prospective mathematics teacher students use mathematical concepts in solving birthday cake problems; and 2) to describe the flow of mathematical thinking processes of first-year prospective mathematics teacher students in solving birthday cake problems.

RESEARCH METHODS

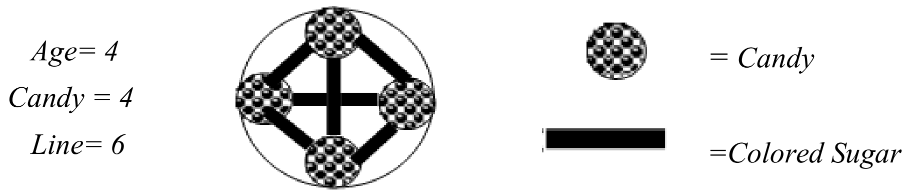
This research uses qualitative research, which aims to analyze student's mathematical thinking processes in solving problems in the context of birthday cakes. Mohajan, (2018) stated that a qualitative approach makes it possible to explore and carry out in-depth analysis. The research design used is a case study. According to (Creswell & Creswell, 2018), one of qualitative approaches that specifically discusses certain cases in a real world context is a case study. The subjects in this research were students in the 2nd semester of the 2023/2024 academic year. The instruments used in this research were birthday cake context questions and interview sheets. Data collection was carried out by giving a test to investigate students' mathematical thinking processes and an interview to clarify students' answers to the birthday cake problem. The students' answers were analyzed based on the indicators of the mathematical thinking process proposed by (Stacey, 2007).

The procedures taken were as follows: 1) Data reduction, which involved the process of collecting and selecting the data used; the results of the mathematical thinking process tests completed by the students were selected and grouped based on the mathematical concepts used to determine the solution to the birthday cake problems. The students' answers could be grouped into five concepts that could be used in solving these birthday cake problems, namely the concept of number patterns, the concept of combinations, the concept of series, the concept of quadratic functions, and the concept of diagonals in geometric shapes. 2) Data presentation, which involves systematically and organized presenting the reduced data with a clear relationship pattern; at this stage, the data grouped by the form of the concepts used were described based on the indicators of the mathematical thinking process. Thus, the students' mathematical thinking processes could be clearly organized. 3) Drawing conclusions, where the data presented were then concluded or interpreted; at this stage, conclusions were drawn about the forms of mathematical concepts that students could use in solving the birthday cake problems, and the mathematical thinking processes formed by students based on the indicators of the mathematical thinking process (Moleong, 2012; Sari, 2021) .

RESULTS AND DISCUSSION

The analysis of the mathematical thinking process found in this research is examining the mathematical concepts used by students in solving birthday cake problems and the mathematical thinking processes used based on four indicators of mathematical thinking abilities of first year prospective mathematics teacher students. The problem with the birthday cake provided is as follows:

My niece, Andrea, was decorating a cake for her daughter's birthday. Her daughter was four, so she placed four candies around the top of the cake and connected all the candies to each other with colored icing, it looked quite pretty! I started thinking about what the cake would look like when the daughter was 3, or what it would look like next year. Appearance of birthday cake from above



Actually, I want to know how many lines I should draw to connect the candy on the cake for all ages. Can you identify and explain how to calculate the number of colored icing lines needed to connect candies on a cake when aged?

The results of the mathematical thinking process tests utilizing the birthday cake problem provided data on various mathematical concepts used by students to solve the problem. The following are the results of the mathematical thinking process tests of 71 students in solving the birthday cake problem.

Table 1. Results of the Mathematical Thinking Process Test of Students Using the Birthday Cake Problem.

No	Mathematical Concepts Used	Emerging Indicators of the Mathematical Thinking Process	Number of Students
1	Concept of Number Patterns	Specialization,Predicting,Verifying	39 Students
2	Concept of Combinations	Predicting, Generalizing, Verifying, and Specialization	26 Students
3	Concept of Series	Predicting, Generalizing, Verifying, and Specialization	6 Students
4	Concept of Quadratic Functions	-	-
5	Concept of diagonals in plane shapes	-	-

Based on the results of student answers, there are three mathematical concepts used in solving birthday cake problems. The three concepts found are the concept of number patterns, the concept of combination, and the concept of series. Researchers found two other concepts in solving this birthday cake problem. Two other concepts that researchers discovered were the concept of quadratic functions and the concept of diagonals of flat shapes. The concepts used by students in solving the birthday cake problem are as follows:

The concept of number patterns

39 students used the concept of number patterns. An example of one of the answers given by students is in figure 1 below:

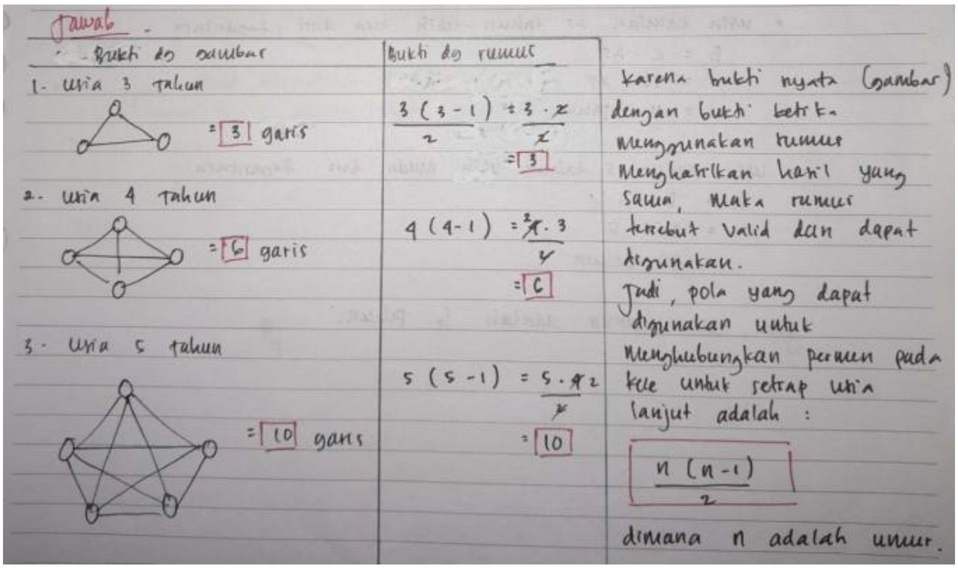


Figure 1.Student answers using the concept of Number Patterns

Figure 1 shows that first, students found data, namely the number of colored icings for several ages. The first data determined was the amount of colored icing for 3 year-olds. Based on the question information, students draw three candies and the colored sugar coating that connects each candy. After connecting everything, students found three colored icings written with “3 lines.” Students wrote “3 lines” as a word to represent three colored layers of sugar. In the same way, at the ages of four and five, students found "6 lines" and "10 lines", which means that at the age of four, they found six layers of colored sugar and at the age of five, they found ten layers of colored sugar. In the second step, students express the number 3 with the product of. $\left(\frac{3(3-1)}{2}\right)$ Likewise, for ages four and five, students stated the numbers six and ten in the form of the previous pattern. Even though when writing patterns at the ages of four and five, they do not write the divisor 2, in the next step, students include the divisor 2 so that they get the results 6 and 10. In this step, students use data to carry out mathematical thinking processes, namely specialization, guessing and convincing. The specialized thinking process carried out by students is to form sequential data, namely for ages three, four, and five. The guessing process that is carried out is to form a number pattern to get the number of colored sugar layers for each age three, four, and five-year-olds. The convincing thought process is to substitute each piece of data into the expected pattern, which produces a statement that is true for three years, four years, and five years. In line with the results of research conducted by (Adegoke, 2015), it shows that the convincing thinking process requires students to develop self-confidence so that they can solve mathematical problems effectively.

The third step taken by students is to draw conclusions based on data obtained by drawing and evidence using the number pattern formula, which always produces a correct statement, so students conclude that the number of colored sugar coatings for various ages is expressed by $\left(\frac{n(n-1)}{2}\right)$ This process is a thinking process generalization, namely obtaining general conclusions about the relationship between age, the number of candies and the number of sugar coatings connected to each candy.The ability of students to connect various conditions found in problems to form a solution should be continuously practiced. Possessing the capability to offer solutions in a variety of different ways, including novel approaches, will aid students in addressing everyday problems they may encounter in their future professional lives (Demirtaº & Batdal Karaduman, 2021).

The steps described by this student are the steps most widely used in using the concept of number patterns from 39 students. Some other students explained it in a non-systematic way, but still in the same direction.

The concept of combination

26 students used the combination concept. An example of one of the answers given by students is in **Figure 2**.

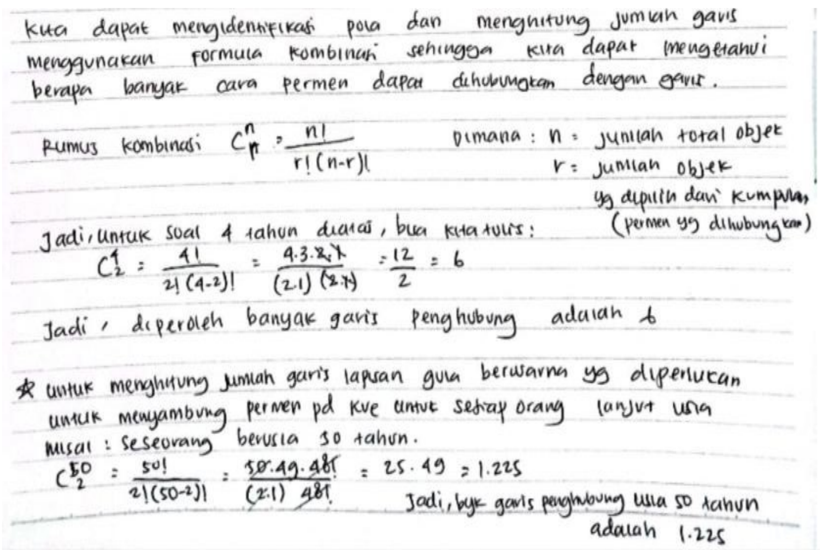


Figure 2. Student answers using the concept of combination

Figure 2 shows that students concluded that the combination formula could determine the number of colored icings for each age. Based on interviews, students stated that they used the combination formulations because the problem of birthday cakes has the characteristic of combination use. The characteristic mentioned is the number of ways to take two candies from the *n* available candies, where *n* candies represent n ages. The number of ways to take two candies from *n* candies is the number of colored layers of sugar formed. Students start by writing the combination formulas . $C_r^n = \frac{n!}{r!\,(n-r)!}$ *n* states the number of candies or age and r states the number of candies involved in forming one colored layer of sugar, namely 2. Next, students determine the result to C_2^4 determine the amount of colored icing at 4 years of age. The results obtained are the same as the information contained in the birthday cake problem. As a result, students are confident that the combination formula can answer the questions regarding birthday cakes. More specifically, students show answers to the number of colored icings for old age when they are 50 years old. The result obtained is 1,225 colored icings. The number 50 is a number that is considered to represent a statement for old age. This condition indicates that mathematical thinking skills can continuously develop in line with the problems given to students. Providing students with opportunities to independently develop their mathematical thinking skills can encourage them to take responsibility for their own learning (Güner & Erbay, 2021).

The mathematical thinking processes involved in these students are guessing, generalizing, convincing and specializing. The thought process of guessing occurs in the process of determining the combination formula, as the formulation determines the number of colored sugar layers. As obtained from the interview results, this determination is based on students' experience in using the combination concept. In the generalization thinking process, students determine values $r = 2$ based on the number of candies selected to form one layer of colored icing and *n* as the amount of candy available. This generalization process occurs because of the student's ability to process and organize the information contained in the problem so that a thinking process is formed (Izzatin et al., 2020). In the convincing thinking process, students take the available information on the birthday cake problem, namely that on their fourth birthday they get four candies and 6 colored icings. By applying the combination formulation that was formed at the age of four, students have confidence that the combination formulation is correct. At the end of the thinking process, students specialize in the specified combination formulation, namely by setting the age of 50 years as old age.

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June 2025, 11(1): 1-15

The concept of series

Students use the series concept with as many as 6 students. An example of one of the answers given by students is in **Figure 3**.

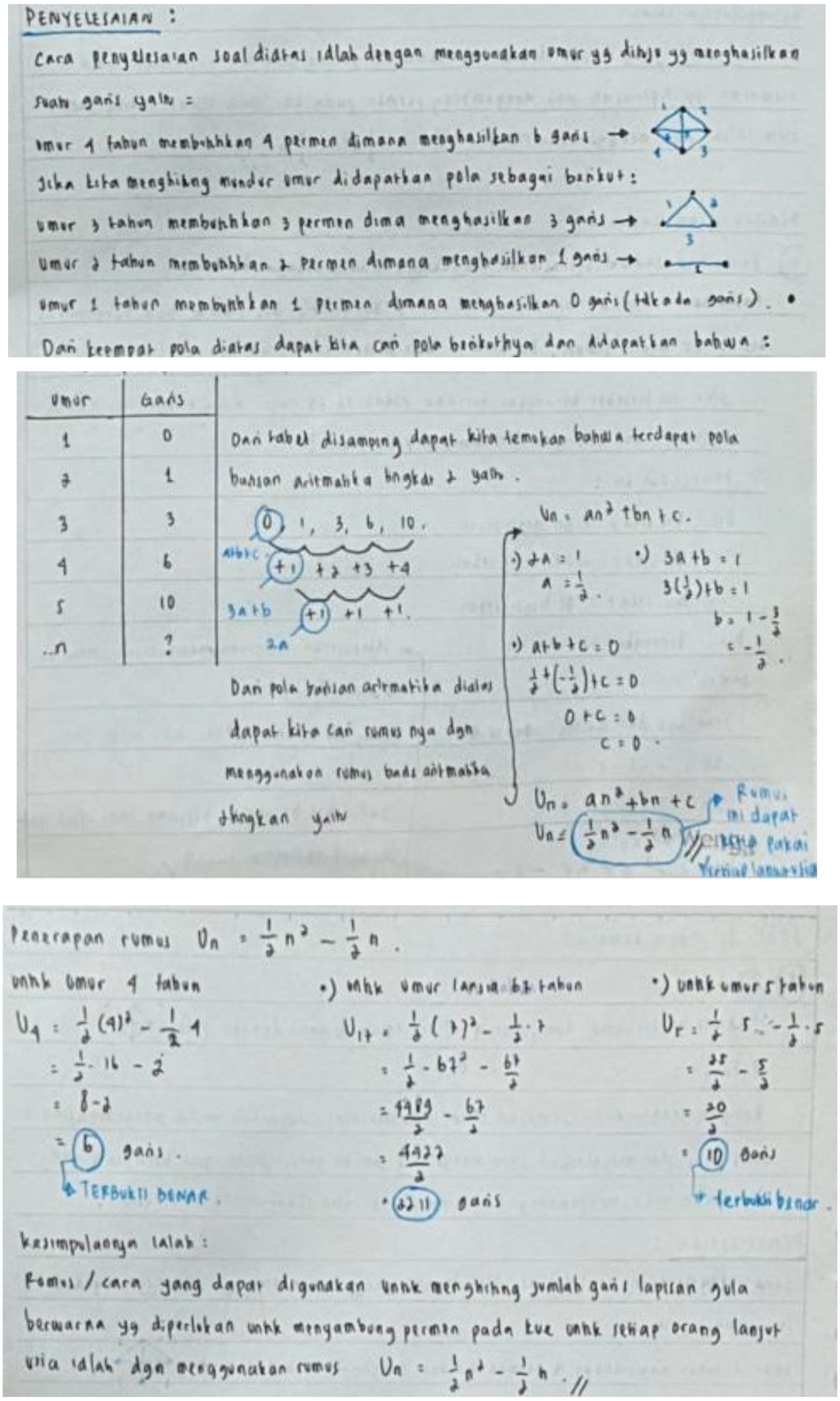

Figure 3. Student answers using the concept of combination

Figure 3 shows that students look for further data based on information on the birthday cake problem. The data found is for 1-year-olds producing 0 lines. The lines in question are colored icing. For 2 years old, it produces 1 colored icing. For 3-year-olds, it produces 3 colored icings. For 5-year-olds, it produces 10 colored icings. Next, a series is formed, which finally results in an arithmetic series at the first level. Based on the general formula for level 2 series and determining the value of each variable, the general formula for the series is obtained: . $u_n = \frac{1}{2}n^2 - \frac{1}{2}n$ Next, students test the formula to u_n data 4 and 5. The results show that u_n was proven correct for the test results. By taking the number 67 as an advanced age, the student obtained the result that 2211 layers of colored sugar were formed. At the end of the student's answer, they concluded that the number of colored icings formed on the birthday cake for eac age could be determined using a series formula u_n that has been obtained.

The mathematical thinking processes involved in these students are specialization, guessing, generalization, and convincing. The specialization thought process begins by counting the number of colored icings for the ages of one, two, three, four, and five. Based on the data obtained, students carry out a conjectural thinking process, namely by conjecturing that the data forms a series. This data is used to form a generalization process by applying the concept of series. By analyzing the series formula, the answer is obtained to determine the number of colored sugar layers. Furthermore, students still need a convincing thought process from the generalization results obtained, namely by trying statements for the ages of four and five. The result obtained is that the statement is proven to be true. The mathematical thinking process conducted by students demonstrates that mathematical thinking can be developed, thereby creating self-awareness among students in achieving learning objectives (Dunphy, 2010; Simon & Cox, 2019). This is consistent with previous research indicating that by combining various processes, students have developed their individual mathematical thinking processes.

Three students out of 68 gave alternative answers, namely using the concepts of combinatory and series. There is no difference in the thinking process from the one described above. The interview results showed that students did this to ensure that the results obtained were correct, even though the concepts were different. It shows that by using both of concepts, the answers are equal. Students' ability to determine alternative concepts in solving problems shows the development of students' mathematical thinking processes. Diversity in conveying ideas or solutions to a problem is fundamental to the thinking process (Minarti et al., 2017).

Two other concepts that researchers discovered were the concept of quadratic functions and the concept of diagonals of flat shapes. However, these two concepts were not found in the thinking processes conducted by the students. The concept of a quadratic function is obtained by the following procedure:

For example, the age at old age is n. There are n-1 colored sugar lines connected to the first candy of another candy. Likewise, there are n-1 colored sugar lines connected to the 2nd, 3rd, 4th candy. Since each colored sugar line is connected to exactly 2 candies, the number of colored sugar lines is equal to half the number of colored sugar lines connected to each candy. The result can be written as a function of n, $f(n) = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$ with n natural numbers. Meanwhile, the diagonal concept of flat shapes is solved using the following procedure:

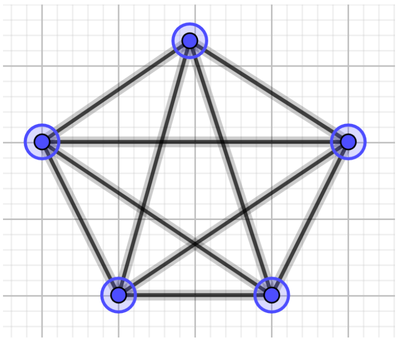

Figure 3.4 The example of a polygon and its diagonals

By relating the birthday cake problem to the number of diagonals in the polygon, we can determine the number of colored sugar lines by adding the number of diagonals in the polygon plus the number of sides. By using the formula for the number of diagonals in the polygon $\frac{n(n-3)}{2}$ and the number of sides in the polygon is n, and then the number of colored sugar lines for age n is equal to $\frac{n(n-3)}{2}$ add n. As a result,

$$\frac{n(n-3)}{2} + n = \frac{n(n-3)+2n}{2} = \frac{n^2-n}{2}$$

This is condition shows that students still need to develop mathematical thinking processes that can train them to explore broader concepts in solving this problem. Based on the mathematical thinking process carried out by students in solving birthday cake problems, it can be found that each mathematical concept used has a different sequence of mathematical thinking processes. This condition is in line with research results that show that mathematical thinking in solving problems can be done by choosing the right solution strategy and doing it in varied ways, as well as the level of knowledge possessed (Ersoy & Güner, 2015). For this reason, as prospective mathematics teachers, students must continue to be supported to develop their abilities through familiarity with mathematical thinking processes.

CONCLUSIONS

The results of the research show that the mathematical thinking process used by first -year prospective mathematics teacher students in solving birthday cake problems involves three mathematical concepts: the concept of number patterns, the concept of combinations, and the concept of addition in arithmetic series. The flow of the mathematical thinking process used by students includes visualizing the birthday cake problem, then observing the specifics of the problem to derive generalizations and proofs. Using the concept of number patterns, the mathematical thinking process involves specialization through drawings, making predictions based on created patterns, and iterating predictions until a conclusion is reached. Using the concept of combinations, the mathematical thinking process starts with predictions, forming generalizations, and proving these generalizations to gain confidence through specialized proof. Using the concept of series, the process begins with specialization, making predictions based on specific cases, forming generalizations, and providing proof as a form of verification. For future researchers, it is important to optimize students' mathematical thinking processes so that they can develop test instruments encompassing multiple concepts and lines of thought, enabling them to assess student's mastery of various concepts. It is hoped that this approach will encourage students to explore as many solution pathways as possible using various understood concepts.

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